



Figure 1: Spherical trigonometry

## Approximating the Sun elevation

The Sun elevation as a function of time can easily be approximated from max/min solar elevations, if we only model earth rotation effects. The resulting precision is about 5 minutes.

### Solar elevation

From figure 1, using the *cosine rule for sides* from *spherical trigonometry* gives,

$$\begin{aligned}\cos(z) &= \cos(s) * \cos(l) + \sin(s) * \sin(l) * \cos(2\pi t) \\ z &= \frac{\pi}{2} - e\end{aligned}$$

where  $e$  is the elevation and the solar time,  $t$ , is the fraction of day starting at noon. We assume constant Sun position,  $s$ , and observer position,  $l$ , which gives us the following relationship between the elevation and solar time,

$$\sin(e) = A + B \cos(2\pi t) \quad (1)$$

where  $A$  and  $B$  are constant (depend on  $s$  and  $l$ ).

### A and B from solar elevation

Let us assume that we know the max and min solar elevation,  $e_{max}$  and  $e_{min}$ . We may write,

$$\begin{aligned}\sin(e_{max}) &= A + B \\ \sin(e_{min}) &= A - B \\ A &= \frac{\sin(e_{max}) + \sin(e_{min})}{2}\end{aligned} \quad (2)$$

$$B = \frac{\sin(e_{max}) - \sin(e_{min})}{2} \quad (3)$$

### Nautical twilight

The time of  $-6$  degree elevation (Nautical twilight start/end) is found using 1, 2 and 3.

$$\begin{aligned}\sin\left(-6 \cdot \frac{2\pi}{360}\right) &= A + B \cos(2\pi t) \\ t_0 &= \frac{1}{2\pi} \arccos\left(\sin\left(\frac{-6 \cdot \frac{2\pi}{360} - A}{B}\right)\right)\end{aligned}$$

Nautical twilight starts,

$$\begin{aligned}t &= N + t_0, \\ N &= 0, 1, 2, 3, \dots\end{aligned}$$

Nautical twilight ends

$$\begin{aligned}t &= N - t_0, \\ N &= 0, 1, 2, 3, \dots\end{aligned}$$