Approximating the upper limb elevation

The Apparent upper limb is the first part of the Sun that you will observe at Sunrise. At sunrise the Sun that you see is actually below the horison, but the atmosphere bends the light so that it becomes visible above the horizon. This atmospheric light bending effect is called *refraction*, as illustrated in figure 1.
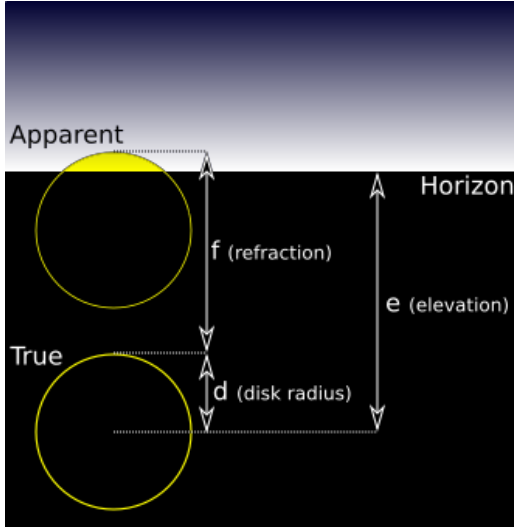

Figure 1: Upper limb elevation.

The elevation, e, indicates the true position of the solar disk center. Note that when the elevation is negative, the true Sun disk center is below the astronomical horizon. The disk radius, d, is the angular extent of the true solar disk. The refraction, f is the angular lifting of the Sun due to the bending of light in the atmosphere. This varies with the elevation and atmospheric conditions and is about 0.58 degrees near the horizon. In astro-api, it is nominally set to $34/60$ degrees at the horizon.

Upper Limb elevation

The upper limb elevation, e_{upper}, is given by

$$
\begin{aligned}
e_{upper} &= e + d \\
d &= \frac{r}{R}(rad) = \frac{r}{R} \cdot \frac{180}{\pi}(deg)
\end{aligned}
$$

where d is the angular radius of the body, and r is the actual radius, while R is the distance to the body (range). For the Sun, astro-api uses $r = 682500(km)$ and for the Moon, $r = 1738(km)$. The typical angular radius of the Sun is $0.26 - 0.27$ degrees, and for the Moon 0.26, degrees.

Rise and set

Rise and set occurs when the upper limb elevation is equal to the negative refraction, f,

$$
\begin{aligned}
e &= -(f + d) \\
f &= \frac{34}{60}(deg).
\end{aligned}
$$

Note that twilight criteria are relative to the (true) elevation of the Sun disk center, without taking refraction effects into account.